ERRATUM TO "AMALGAMATION OF CELLULAR AUTOMATA" (MAY 2008)

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Abstract. The paper "Amalgamation of Cellular Automata" accepted at *JAC 2008* contains 2 errors: a statement with a wrong proof and an example of amalgamation operation which doesn't fit into the precise setting of the paper. At the time of writing this short erratum, the author has no proof of the statement and no idea on how to generalise the setting of the paper to capture the example without breaking the proofs.

However, main results of the paper are not affected. Moreover, the following erratum provides a weaker statement with a correct proof and a slightly less general amalgamation operation to replace the buggy one.

1. Errors

The paper contains two errors:

(1) the example amalgamation operation Γ_a (definition 3.3) is flawed since it violates condition 2 of definition 2.4 ($\Gamma_a(F, G)$ contains less automata if F and G possess many sub-automata than if they don't possess any);
(2) proof of proposition 5.2 is wrong since the cellular automaton δ_H constructed is not captive as it should be (both the first and the second component of the produced state are always present in the neighbourhood but the state itself isn't necessarily present).

2. Workarounds

First, the operation Γ_a can be replaced everywhere by $\Gamma_{\mathcal{K}}$. Propositions 4.3 and 5.3, corollary 4.5 and theorem 5.4 together with their proofs can be kept unchanged after the syntactical replacement.

Second, proposition 5.2 can be replaced by the following one without affecting theorem 5.4 (which uses it).

Proposition 2.1. *There exist intrinsically universal cellular automata in families $\mathcal{MAJ}$ and $\mathcal{MIN}$.*

Key words and phrases: cellular automata, amalgamation, density, zero-one law, universality.

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Proof sketch. The proof is almost identical for $\mathcal{MAJ}$ and $\mathcal{MIN}$ and we treat only the $\mathcal{MAJ}$ case. We will show that for any CA F with two states, there is a $G \in \mathcal{MAJ}$ (with more states and a larger radius) such that $F \preceq G$. The key idea is to do the simulation over a set of configurations such that two occurrences of a same state can never be seen in the same neighbourhood of G. On such configurations, the majority constraint reduces to the captivity constraint.

More precisely, if F has a radius r, then G has a radius $2r$ and state set

$$Q = \{0,1\} \times \{0,\ldots,4r+1\}$$

. For i between 0 and $4r+1$, an i-bloc is one of the two 2-states words:

- $b_i(0) = (0,i)\cdots(1,i)$, or
- $b_i(1) = (1,i)\cdots(0,i)$.

an i-bloc can encode one bit of information or, equivalently, a state of F.

Then, a configuration c of F is encoded by the configuration $\Phi(c)$ of G made of a succession of i-blocs:

$$\cdots b_{4r+1}(c_{-1}) \cdot b_0(c_0) \cdot b_1(c_1) \cdots b_{4r+1}(c_{4r+1}) \cdot b_0(c_{4r+2}) \cdots$$

The behaviour of G is designed in such a way that $\Phi\big(F(c)\big) = G\big(\Phi(c)\big)$. This can be done with $G \in \mathcal{MAJ}$ since:

- encoded configurations $\Phi(c)$ have the property above (two occurrences of a state cannot be seen in a same neighbourhood) and
- an i-bloc $b_i(0)$ can be turned into $b_i(1)$ (and reciprocally) by a captive transition since it suffices to invert the occurrences of 2 states (precisely, $(0,i)$ and $(1,i)$).

Details of the construction are left to the reader. ∎