ERRATUM TO THE PAPER: Communication Complexity and Intrinsic Universality in Cellular Automata*

E. Goles[a], P.-E. Meunier[c], I. Rapaport[b], G. Theyssier[c,*]

[a] Facultad de Ingenieria y Ciencias, Universidad Adolfo Ibáñez, Santiago, Chile
[b] DIM, CMM (UMI 2807 CNRS), Universidad de Chile, Santiago, Chile
[c] LAMA, Université de Savoie, CNRS, 73 376 Le Bourget-du-Lac Cedex, France

Abstract

Proofs of Propositions 6 and 8 of the paper *Communication Complexity and Intrinsic Universality in Cellular Automata* are formally incorrect. This erratum proves weaker versions of Propositions 6 and 8 and a stronger version of Proposition 9 which are sufficient to get the main results of the paper (Corollary 2) for PREDICTION and INVASION problems. For problem CYCLE, we only prove a weaker version of Corollary 2, essentially replacing a condition of the form '$f \in \Omega(n)$' by '$f \notin o(n)$'. All other statements of the paper are unaffected.

1. Comparison relation

In subsection 4.1 of the paper, a relation $\prec$ between functions from $\mathbb{N}$ to $\mathbb{N}$ is defined. It should be replaced by the following:

Definition 1. $\phi_1 \prec \phi_2$ *if there are non-constant affine functions* $\alpha, \beta, \gamma, \delta$ *from* $\mathbb{N}$ *to* $\mathbb{N}$ *such that* $\alpha \circ \phi_1 \circ \beta \leq \gamma \circ \phi_2 \circ \delta$.

By non-constant affine function, we mean a function of the form $n \mapsto \alpha n + \beta$ for some $\alpha > 0$. From now until the end of this erratum, the notation $\prec$ refers to the above definition.

Remark. If a function ϕ is $\prec$-greater than the identity $n \mapsto n$ then $\phi \notin o(n)$. However, it is not generally true that $\phi \in \Omega(n)$.

Lemma 1. *Let f be the identity function ($f(n) = n$). Let F be any CA and let $g = \mathrm{CC}\left(\textsc{Pred}_F\right)$ and let $h = \mathrm{CC}\left(\textsc{Inv}_F^u\right)$ for some word u. Then we have:*

- *if $f \prec g$ then $g \in \Omega(n)$;*
- *if $f \prec h$ then $h \in \Omega(n)$;*

*Corresponding author (guillaume.theyssier@univ-savoie.fr)
*Partially supported by programs Fondap and Basal-CMM, Fondecyt 1070022 (E.G) and Fondecyt 1090156 (I.R.).

Proof. From the definition of $\prec$, if a function ϕ verifies $f \prec \phi$ then:

$$\exists n_0, \alpha > 0, \beta > 0 \text{ such that } \forall n \geq n_0, f(\alpha n) \geq \beta n.$$

Now we claim that g has the following property:

$$\exists k_0, \forall k \geq k_0, \exists C_k \text{ such that } \forall n \geq k, g(n) \leq C_k g(n-k).$$

This property is sufficient to prove that $g \in \Omega(n)$. This property is true for $k_0 = 2r+1$ since, if w is a word of size n and $k \geq k_0$, $\textsc{Pred}_F(w)$ can be computed from the list of $\textsc{Pred}_F(w_i)$ (with $0 \leq i \leq k$) where w_i is the subword of w of length $n-k$ starting at position i.

To finish the proof it is sufficient to notice that h is an increasing function: indeed, the problem $\textsc{Inv}_F^u$ restricted to inputs of size n is a sub-problem of $\textsc{Inv}_F^u$ restricted to inputs of size $n+1$ if we add the letter number $n+1 \bmod |u|$ of u at the end of each input of size n. □

2. Proposition 6 and 7

Proposition 6 and 7 are true using the new definition of $\prec$ and are proved without changing anything in the original proofs.

3. Proposition 8

Proposition 8 is true if we restrict the simulation relation $\preccurlyeq$ to a weaker relation where composition with shifts are not allowed. Precisely, denote by $F \preccurlyeq_w G$ if there are parameters m, m', t, t' such that $F^{<m,t,0>} \sqsubseteq G^{<m',t',0>}$. If we replace '$F \preccurlyeq G$' by '$F \preccurlyeq_w g$' in the statement of Prop. 8, then it becomes correct with exactly the same proof.

4. Proposition 9

Let F be the CA used to prove item 3 of Proposition 9. In fact, F has the following stronger property:

$$\forall t, \forall z, \forall k \geq 1, \mathrm{CC}\left(\textsc{Cycle}^k_{F^{<1,t,z>}}\right) \in \Omega(n)$$

Informally, not only F is hard for the cycle problem but any finite composition of F and shifts. To show this it is sufficient to consider inputs suggested by the proof with the additional restriction that $x_1 = 1$, $x_2 = 0$ and $y_1 = 0$ and $y_2 = 1$. The problem $\textsc{Disj}$ can still be encoded into such inputs and the presence of at least one '1' is granted in both F_1 and F_2 layer. Therefore, whatever the composition of F and shifts we take, we will get a $\Omega(n)$ rotation on at least one of the two components in case of disjoint inputs ($\bigwedge_{i=1}^n \neg(x_i \wedge y_i) = 1$).

5. Corollary 2

Item 3 of Corollary 2 is false. We can have a universal CA for which the CYCLE problem is trivial as soon as the input period is odd: just add a layer that checks that two states (say black and white) are alternating everywhere and produces a spreading state as soon as two consecutive black cells or two consecutive white cells are in the neighbourhood.

Item 3 should be replaced by the following:

$$\text{there exists } k \text{ s.t. } \mathrm{CC}\left(\textsc{Cycle}_F^k\right) \notin o(n).$$

With all previous modifications, Corollary 2 can be proved as follows.

Proof. Items 1 and 2 follow directly from Lemma 1 of this erratum and Propositions 6, 7 and 9.

For item 3, denote by G the CA having property of item 3 of Proposition 9. By definition of $\preccurlyeq$, since $G \preccurlyeq F$ (F is universal), we have

$$G^{<m,t,z>} \sqsubseteq F^{<m',t',0>}$$

for some parameters m, m', t, t', z (informally, it is always sufficient to use shifts only in the simulated CA). Therefore, we have $G^{<1,t,z>} \preccurlyeq_w F$. By Proposition 9 (item 3 modified as above) and Proposition 8, we deduce that there is k such that $\mathrm{CC}\left(\textsc{Cycle}_F^k\right)$ is $\prec$-above some $\Omega(n)$ function. We finally deduce that: $\mathrm{CC}\left(\textsc{Cycle}_F^k\right) \notin o(n)$. □

Acknowledgment

We would like to thank Raimundo Briceño Dominguez for pointing us the problems in the original proofs and suggesting some corrections.