Clandestine Simulations in Cellular Automata

JAC 2010

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December 16th 2010

Overview of the talk

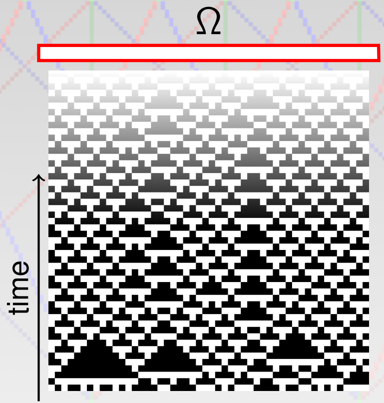
1. Limit sets and column factors
2. Sub-automata and factors
3. A word about proofs
4. Open problems

1 Limit sets and column factors

2 Sub-automata and factors

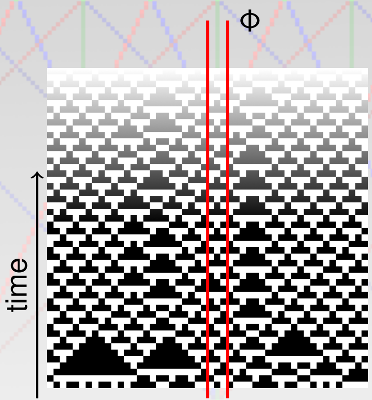
3 A word about proofs

4 Open problems



Limit set

Asymptotic observation



Column factor

Unprecise observation

Definitions

- F a cellular automaton

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- **limit set:** $\Omega_F = \bigcap_t F^t(Q^{\mathbb{Z}})$
  - *upper-bound:* co-r.e. language
  - *varying complexity:* from **finite** to **non-recursive**

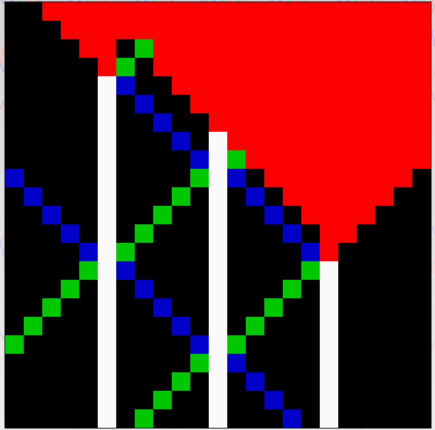
Definitions

- F a cellular automaton

- **limit set:** $\Omega_F = \bigcap_t F^t(Q^{\mathbb{Z}})$
  - *upper-bound:* co-r.e. language
  - *varying complexity:* from **finite** to **non-recursive**

- **column factor of width k:** $\Phi_F = \left([F^t(Q^{\mathbb{Z}})]_k\right)_t$
  - *upper-bound:* context-sensitive language
  - *varying complexity:* from **finite** to **non-context-free**

Example I

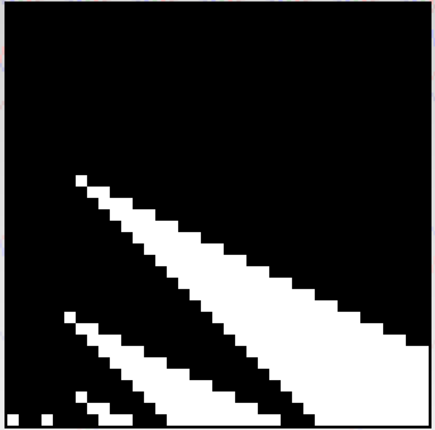


- left/right particles
- **symmetric** bounce on walls
- spreading state otherwise

Exercise

Show that it has a non-sofic limit set.

Example II



$$F(x)_i = x_{i+1}x_{i+2}$$

(example from P. Kůrka)

Exercise

Show that it has a non-sofic column factor.

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Subautomaton

- $F \sqsubseteq G$

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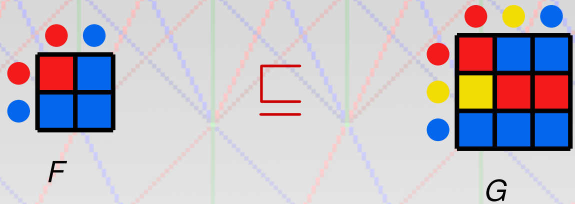
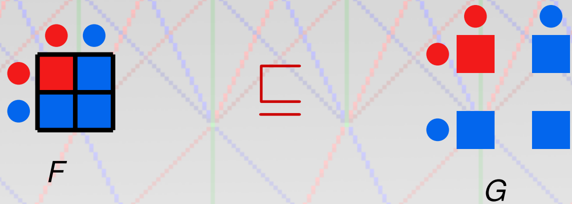


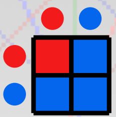
Subautomaton

- $F \sqsubseteq G$



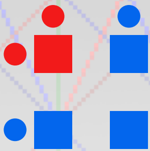
injection $\iota : Q_F \to Q_G$ *with* $\iota \circ F = G \circ \iota$

- $F \sqsubseteq G$



F

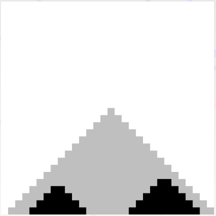
$\sqsubseteq$



G

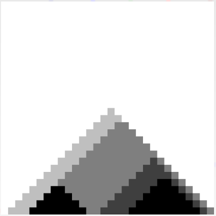
injection $\iota : Q_F \to Q_G$ with $\iota \circ F = G \circ \iota$

- example:



MAX with 3 states

$\sqsubseteq$



MAX with 5 states

Local Factor

- $F \trianglelefteq G$

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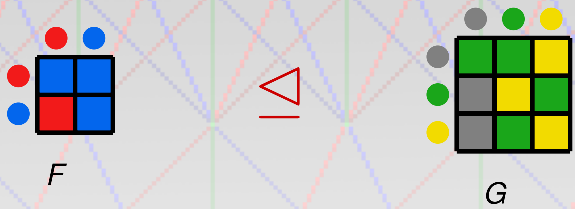
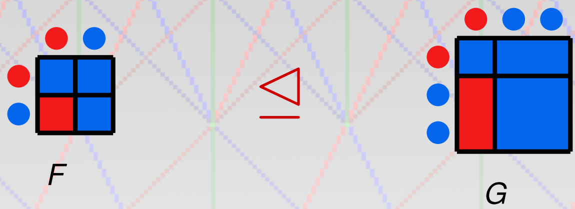


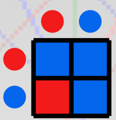
- $F \trianglelefteq G$



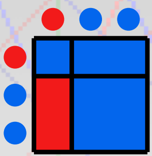
surjection $\pi : Q_G \to Q_F$ *with* $\pi \circ G = F \circ \pi$

Local Factor

- $F \trianglelefteq G$



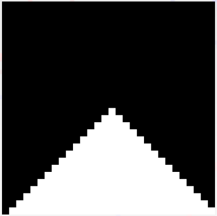
$\trianglelefteq$



F G

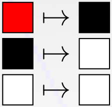
surjection $\pi : Q_G \to Q_F$ *with* $\pi \circ G = F \circ \pi$

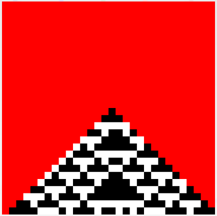
- example



MAX with 2 states

$\trianglelefteq$





54 + spreading state

Some facts

- If $F \trianglelefteq_\pi G$ then (compacity argument)

$$\Omega_F = \pi(\Omega_G)$$
$$\Phi_F = \pi(\Phi_G)$$

  - ***complexity increases from F to G***

Some facts

- If $F \trianglelefteq_\pi G$ then (compacity argument)

$$\Omega_F = \pi(\Omega_G)$$
$$\Phi_F = \pi(\Phi_G)$$

► ***complexity increases from F to G***

- If $F \sqsubseteq G$ then

$$\Omega_F \subseteq \Omega_G$$
$$\Phi_F \subseteq \Phi_G$$

► *does complexity increase from F to G?*

Clandestine simulation

Complexity can be hidden in subautomata

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Complexity can be hidden in subautomata

Theorem

Let F be any CA, then

1. there exists G with $F \sqsubseteq G$ and Φ_G **of finite type**
2. there exists G with $F \sqsubseteq G$ and Ω_G **NLOGSPACE-rec.**

Clandestine simulation

Complexity can be hidden in subautomata

Theorem

Let F be any CA, then

1. there exists G with $F \sqsubseteq G$ and Φ_G **of finite type**
2. there exists G with $F \sqsubseteq G$ and Ω_G **NLOGSPACE-rec.**

Corollary

There are instrinsically universal CA with a 'simple' limitset or a 'simple' column factor.

1 Limit sets and column factors

2 Sub-automata and factors

3 A word about proofs

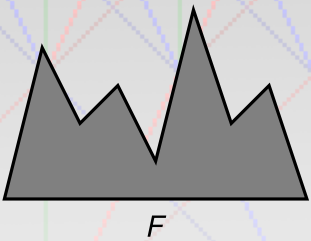
4 Open problems

General idea: flooding

Resia Lake, Italy

General idea: flooding

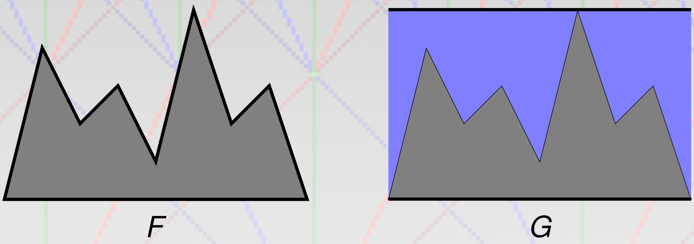
- $F \sqsubseteq G$



observation $\equiv$ *surface*

General idea: flooding

- $F \sqsubseteq G$

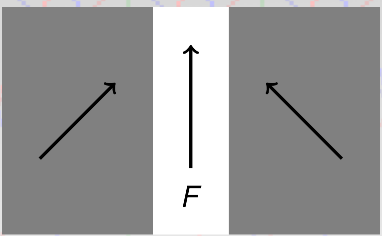


$$\textit{observation} \equiv \textit{surface}$$

$$\text{complexity(F)} \;\geq\; \text{complexity(G)}$$

The case of column factor

- $G \approx F \times$transport layer



The case of column factor

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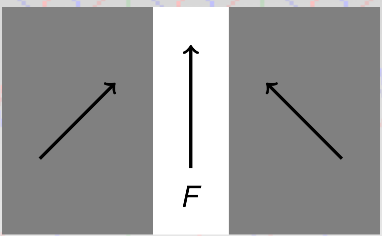


2-approximation of Φ_G

Set of columns which are valid when looking through a window of 2 time steps.

- **Fact:** Φ_G is equal to its 2-approximation
- **Corollary:** Φ_G is an SFT of order 2

The case of limit set

- $G \approx F \cup (F \times S)$

1. **on $F \times S$ component:** identity on F and firing squad on S
2. **when S fires:** switch to F component
3. **on F component:** do like F

The case of limit set

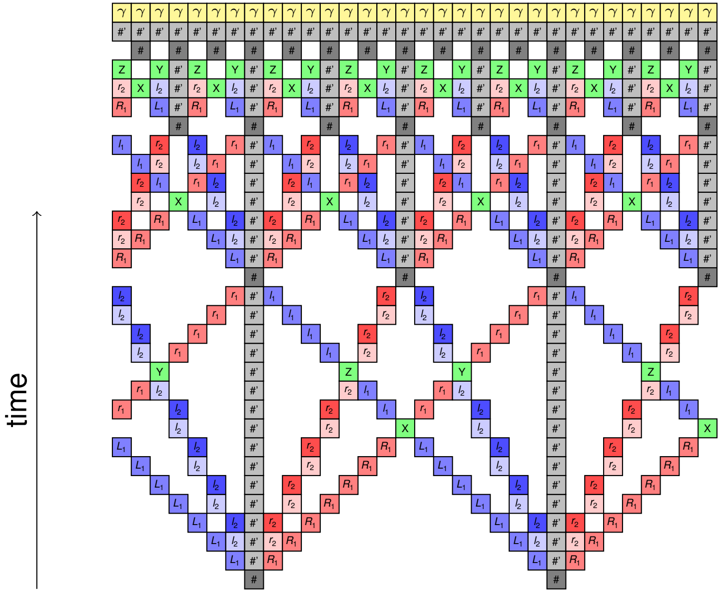
- $G \approx F \cup (F \times S)$
  1. **on $F \times S$ component:** identity on F and firing squad on S
  2. **when S fires:** switch to F component
  3. **on F component:** do like F

- **Fact:** the language of Ω_G is NLOGSPACE
  - **easy part:** the whole F component is in Ω_G

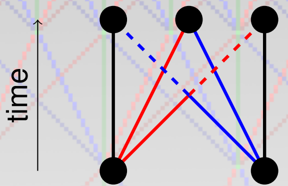
$$\Omega_G \cap Q_F^{\mathbb{Z}} = Q_F^{\mathbb{Z}}$$

  - hard part: what else is in Ω_G?

Jarkko Kari’s Firing squad



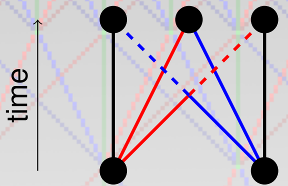
Euclidean reasoning



- valid $\stackrel{def}{=}$ no fire and no spreading state
- valid discrete diagram $D \Rightarrow$ valid Euclidean diagram E with

$$E_{|\mathbb{Z}^2} = D$$

Euclidean reasoning



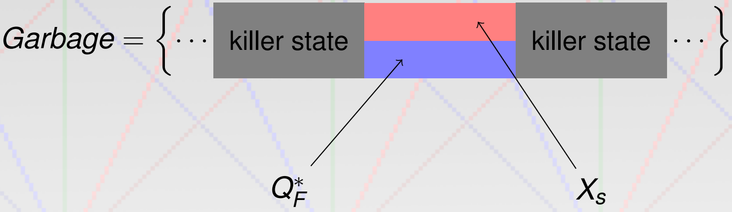
- valid $\stackrel{def}{=}$ no fire and no spreading state
- valid discrete diagram $D \Rightarrow$ valid Euclidean diagram E with

$$E_{|\mathbb{Z}^2} = D$$

- **Proof technique:**
  1. $u \in \Omega_G$?
  2. try to construct an Euclidean history of u
  3. get an Euclidean contradiction
  4. conclude that $u \notin \Omega_G$

Garbage collection

$$\Omega_G = Q_F^{\mathbb{Z}} \cup \textit{Garbage}$$



- X_S: confs with infinite history without killer or fire
- **(technical!) fact:** X_S is NLOGSPACE recognizable

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Open problems

- Intrinsic universal CA:
  - with a SFT limit set?
  - with an injective limit system?
- is there a firing squad CA with an SFT limit set?
- factor maps:
  - is injectivity preserved by factor maps?
  - is there an expansive CA factor of a non-expansive CA?